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Inferring neighbourhood quality with property transaction records by using a locally adaptive spatial multi-level model

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ABSTRACT

Social and physical processes often exhibit both macro-level geographic smoothness – implying positive spatial dependence – and micro-level discontinuities – suggesting implicit step changes or boundaries in the data. However, a simultaneous treatment of the two features in a unified statistical model poses great challenges. This study extends an innovative locally adaptive spatial auto-regressive modelling approach to a multi-level modelling framework in order to explore multiple-scale geographical data. It develops a Bayesian locally adaptive spatial multi-level model that takes into account horizontal global spatial dependence and local step changes, as well as a vertical group dependency effect imposed by the multiple-scale data structure. At its heart, the correlation structures of spatial units implied by a spatial weights matrix are learned along with other model parameters using an iterative estimation algorithm, rather than being assumed to be invariant and exogenous. A Bayesian Markov chain Monte Carlo (MCMC) sampler for implementing this new spatial multi-level model is derived. The developed methodology is applied to infer neighbourhood quality using property transaction data, and to examine potential correlates of neighbourhood quality in Liverpool. The results reveal a complex and fragmented geography of neighbourhood quality; besides an overall smoothness trend, boundaries delimiting neighbourhood quality are scattered across Liverpool. Socio-economics, built environment, and locational characteristics are statistically significantly associated with neighbourhood quality.

1. Introduction

Multi-level modelling has been demonstrated as a useful tool to derive summary statistics for higher-level (or more aggregated) units from outcomes measured for low-level units (e.g. individual). For instance, individual pupils' educational achievement has been used to infer school effectiveness and produce league tables to inform parents' school and residence choices (e.g. Leckie & Goldstein, 2009). Such a model for two levels (or scales) is shown in Eq. 1.

$$y_{ij} = x_{ij}\beta + \theta_j + \epsilon_{ij}, \quad i = 1\ldots,n_j; j = 1, \ldots,J$$

$$\theta_j = z_j\gamma + \zeta_j, \quad \epsilon \sim N(0,\sigma_\epsilon^2); \zeta \sim N(0,\sigma_\zeta^2). \tag{1}$$

If y measures pupils' educational outcomes, x measures pupils' characteristics, and i and j are pupil and school indicators in Eq. (1), θ_j will be derived effectiveness measure for school j. n_j represents the number of pupils in school j and J is the total number of schools. β and γ are vectors of regression coefficients to estimate. ϵ and ζ are model residual terms at the pupil and school levels, assumed to follow independent Normal distributions with variances of ${\sigma_\epsilon}^2$ and ${\sigma_\zeta}^2$, respectively.

Two important advantages pertain to the multi-level model-based estimates. First, there is great flexibility in terms of controlling for pupil-level characteristics (e.g. prior education achievement) and understanding the links of school-level characteristics to effectiveness. Second, the estimates on school effectiveness are reliable because of the borrowing strength from other data points (Goldstein, 2011; Raudenbush & Bryk, 2002). The estimator of θ_j, $\widehat{\theta}_j$, is shrunk to the global mean of y after controlling for pupil-level covariate effects, depending on the magnitudes of ${\sigma_\zeta}^2$, ${\sigma_\epsilon}^2$ and n_j, thus less subject to sampling uncertainties in particular schools (e.g. Jones, 1991; Leckie & Goldstein, 2009). Model (1) and its variants have also been applied to derive health statistics for aggregated spatial units by using data available at a fine-resolution spatial scale (e.g. Arcaya, Brewster, Zigler, & Subramanian, 2012; Ma, Mitchell, Dong, & Zhang, 2017), and serve as an important approach in the model-based small area estimation literature (e.g. Rao, 2003).

Directly applying standard multi-level models to spatial data using

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spatial groups as the "second (or higher) level" has been shown to be problematic (Arcaya et al., 2012; Bivand, Sha, Osland, & Thorsen, 2017; Dong & Harris, 2015; Dong, Ma, Harris, & Pryce, 2016). A key concern is that geography exists both "vertically," in the sense that observations have regional membership, and "horizontally," in that spillovers between adjacent observations often exist regardless of grouping (Haining, 2003; Owen, Harris, & Jones, 2016). A few spatial extensions on multi-level models have been developed by specifying higher-level residuals as a simultaneously autoregressive (SAR) model (e.g. Dong & Harris, 2015; Lacombe, Holloway, & Shaughnessy, 2014; Lacombe & McIntyre, 2016; Savitz & Raudenbush, 2009), or as a conditional autoregressive (CAR) model (e.g. Dong et al., 2016; Ma et al., 2017). Whilst global spatial auto-correlation or large-scale smoothness pattern in the outcome variable under investigation can be captured, potential local step changes are ignored in the proposed spatially explicit multi-level models mentioned above. Local step changes or boundaries refer to those geographic borders, areas on the opposite sides of which are associated with abrupt changes in outcome values. They need to be taken into account when modelling spatial auto-correlation as it would be inappropriate to assume areas that are separated by boundaries to be correlated as strongly as those that are not.

1.1. Boundary detection and a locally adaptive spatial auto-correlation model

The detection of boundaries in the distributions of geographical variables on its own is the key aim of the areal Wombling literature, originating from Womble (1951). We refer to Jacquez, Maruca, and Fortin (2000) and references therein for various detection algorithms developed for point-referenced and image or gridded spatial data. These techniques are often not based on statistical models, and thus less suitable to deal with sampling uncertainties underlying data of interest (Dean, Dong, Piekut, & Pryce, 2018; Lu & Carlin, 2005). Lu and Carlin (2005) proposed a Bayesian areal Wombling approach where a Bayesian CAR model was applied to data on county-level cancer incidence in Minnesota. This method produces rich model estimates using Markov chain Monte Carlo (MCMC) sampling, and boundaries are then identified by comparing the distributions of expected differences between geographically bordering areas. Given the primary focus of boundary detection in this strand of literature, the identified local step changes or discontinuities are not further used to inform the modelling of spatial auto-regression. Like other domains, the connectivity structure is taken as exogenous, and boundaries are detected on top of it.

In a single-level spatial data context, Lee and Mitchell (2013, 2014) proposed an innovative locally adaptive spatial auto-correlation modelling approach. The key idea of the approach is to model ambivalence between "no correlation" and "no connectivity" in the final model covariance matrix. Spatial correlation is conceptualised as global, with the strength determined by the full-map (or global) spatial autoregressive parameter, but attenuated locally if a boundary is detected. In other words, if a step change is detected between two geographically adjacent areas, the (conditional) correlation between them is constrained to be zero by disconnecting them in the spatial weights matrix, W. In this strategy, both the connectivity structure, the strength of full-map spatial autoregression, and the remaining model parameters are estimated. An appealing feature of the locally adaptive modelling approach is that it contrasts strongly with the conventional treatment of W as exogenously known in the spatial modelling literature (e.g. Anselin, 1988; Haining, 2003).

1.2. Innovation of this study

In this study, we develop a Bayesian locally adaptive spatial multi-level modelling approach, in which both global spatial autoregressive structure, local step changes and the multi-scale data structure are captured. It adapts the locally adaptive spatial auto-regression model in Lee and Mitchell (2013), and extends it to a multi-level modelling framework to investigate multi-scale data. The conceptualisation of spatial auto-regression at the neighbourhood scale (for $\boldsymbol{\theta}$) as a locally adaptive model presents our first methodological improvement on the ongoing development of spatially explicit multi-level modelling approaches. Second, this study extends the locally adaptive spatial CAR model to a SAR model, and in doing so, the proposed methodology allows spillover and feedback effects arising from neighbourhood-scale covariate effects to be captured (detailed below). Lastly, the effects on outcomes of independent variables measured at different scales are distinguished, so their interpretations are more clear. This resonates with the idea that different processes might be operating at different spatial scales, and that outcomes at different scales tend to be influenced by different sets of predictor variables. For instance, pupils' educational outcomes are directly related to individual-level characteristics while school effectiveness ($\boldsymbol{\theta}$) is linked to school-level characteristics and impacts pupils' educational outcome as a composite latent variable.

The locally adaptive spatial multi-level model developed here is implemented by using an iterative algorithm following Lee and Mitchell (2013). In a nut shell, it cycles between estimating model parameters via a Bayesian global spatial multi-level model and updating the spatial weights matrix W, until a convergence criterion is met. Bayesian MCMC samplers are derived to implement the new global spatial multi-level model, which constitutes the core component of the overall algorithm.

The proposed methodology is applied to infer neighbourhood quality based on individual property transaction records in Liverpool, as well as information about neighbourhood characteristics. The derived neighbourhood quality estimate is a composite measure of neighbourhood impact on property prices, net of the property-level covariate effects. Neighbourhood quality, often measured by neighbourhood socio-demographic, ethnic, and locational characteristics, has been linked to property prices in voluminous hedonic or spatial hedonic price studies (e.g. Anselin & Le Gallo, 2006; Dubin, 1992; Lazrak, Nijkamp, Rietveld, & Rouwendal, 2014). Nonetheless, neighbourhood quality, as an abstract influencing factor of property prices, includes various facets of a neighbourhood and cannot be completely characterised by a small set of observable neighbourhood attributes. As such, the study regards neighbourhood quality as a latent variable at the property-level equation but as an outcome variable at the neighbourhood-level equation, explained by a range of observable neighbourhood characteristics. This permits unobservable (to researchers) and unmeasurable factors of neighbourhood quality to be captured through the neighbourhood-level residual vector and, reliable estimates on neighbourhood quality to be learned by exploiting the variations in property prices. The locally adaptive spatial multi-level model is devised for fulfilling this purpose. Neighbourhood quality indicators derived as well as the associated uncertainties are useful in a variety of contexts, both academic and policy related. From a research standpoint, aggregate indices of this kind provide statistical summaries that capture multi-dimensional realities of interest for urban economists, geographers and planners. From a policy point of view, the derivation of aggregate measures of neighbourhood quality or attractiveness can be seen as a vehicle to help translate the outputs of complex and sophisticated models to non-technical audiences who are nevertheless interested in their results. For instance, local authorities and urban policy makers may not be familiar with advanced spatial modelling but can still benefit from accurate measures of neighbourhood attractiveness.

The remainder of this paper is structured as follows. Section 2 describes the locally adaptive spatial multi-level model and its estimation method. Section 3 describes the data and variables used in the study. In Section 4, we present and discuss model estimation results. Section 5 concludes with a brief summary of our findings and discussions on potential limitations of the study.

2. Methodology

2.1. The locally adaptive spatial multi-level model

The locally adaptive spatial multi-level model is built upon a multi-level model with spatial auto-regression or dependency. We specify this as a simultaneous autoregressive process within a typical multilevel structure,

$$\boldsymbol{y} = X\boldsymbol{\beta} + \Delta\theta + \epsilon; \tag{2}$$

$$\theta = \rho W\theta + Z\boldsymbol{\gamma} + \zeta, \quad \epsilon \sim N(0, \sigma_\epsilon^2);\ \zeta \sim N(0, \sigma_\zeta^2), \tag{3}$$

where $\boldsymbol{y}$ is a $N \times 1$ vector of property transaction prices with N being the sample size, X is a $N \times P$ matrix of property-level variables, and Δ is a $N \times J$ random effect design matrix. $\boldsymbol{\beta}$ is a $P \times 1$ column vector of regression coefficients to estimate.

The $J \times 1$ column vector $\boldsymbol{\theta}$ is the neighbourhood-level substantive effect, each neighbourhood's distinct impact on property prices. Neighbourhood quality is itself conceptualised as simultaneously autoregressive in its outcome. The simultaneous autoregressive model of response, sometimes called a "spatial lag model," describes a full pattern of response in the outcome of a spatial process (here, neighbourhood quality, $\boldsymbol{\theta}$) using neighbourhood-level characteristics Z, a $J \times K$ matrix of predictors, that are paired with the $K \times 1$ neighbourhood-level substantive effects γ, which must be estimated.

In this model of the spatial feedbacks in house price and neighbourhood quality, nearby neighbourhoods have an effect on (and are affected by) their surrounding neighbourhoods. By modelling $\boldsymbol{\theta}$ as a spatially-lagged outcome, we embed our belief that neighbourhood quality may (in part) be a function of the desirability of nearby neighbourhoods itself. Since property pricing is a competitive process, we might expect high-quality neighbourhoods to drive up prices in nearby similar neighbourhoods. Likewise, the presence of a low-priced neighbourhood may force sellers in a nearby high-priced neighbourhood to start pricing downwards to compete. We do this in an endogenous specification rather than in a so-called "spatial lag of X" specification (Halleck Vega & Elhorst, 2015) since we believe the outcomes to be mutually-constitutive rather than exogenous based on potential inputs to neighbourhood quality; what is most relevant to the spillover process is the price itself, not the hypothesized drivers of that price. We note, however, that spatially lagged X can be included in Eq. (3) to achieve a so-called spatial Durbin model (e.g. Elhorst, 2010) with no need to adapt the model estimation algorithm derived below. It is also possible to model spatial auto-correlation in the neighbourhood-level residuals (ζ), leading to a spatial error model at the neighbourhood scale (Anselin, 1988; Elhorst, 2010). In this case, the model becomes equivalent to the hierarchical spatial auto-regressive model proposed in Dong and Harris (2015), after inserting Eq. (3) to Eq. (2) and applying minor algebraic manipulation. The implementation of such a model is made available by an open source statistical software package HSAR (Dong, Harris, & Mimis, 2017) in R (R Core Team, 2017).

For this model, W encodes the initial spatial relationships between neighbourhoods. In our study, the entries of W encode geographical proximity: $w_{lk} = 1$ if neighbourhoods (l, k) share a common geographic border, and 0 otherwise. In model estimation, W is usually row-normalised such that the maximum value of the spatial auto-regressive parameter ρ is one. Finally, specifying prior distributions for each model parameter in Eqs. (2) and (3) completes the above Bayesian spatial multi-level model.

The schematic diagram of the above model is illustrated in Fig. 1. In this model, neighbourhood quality (or its effects on property prices) is conceptualised as spatially dependent latent construct, which depends on neighbourhood-level characteristics Z and the random error term $\boldsymbol{\zeta}$. The impacts of Z on property prices are through $\boldsymbol{\theta}$, respecting the well-recognized argument that geographical outcomes might be influenced by varying processes at different scales (Haining, 2003). The diagram reveals an important departure of the methodology developed here from the prior efforts that brings together spatial econometrics and multi-level models (e.g. Dong et al., 2016; Dong & Harris, 2015; Dong, Harris, Jones, & Yu, 2015) where the effects of independent variables, measured at different scales, on an outcome variable are not separated, i.e. the neighbourhood-scale covariates Z are directly related to $\mathbf{y}$ rather than $\boldsymbol{\theta}$.

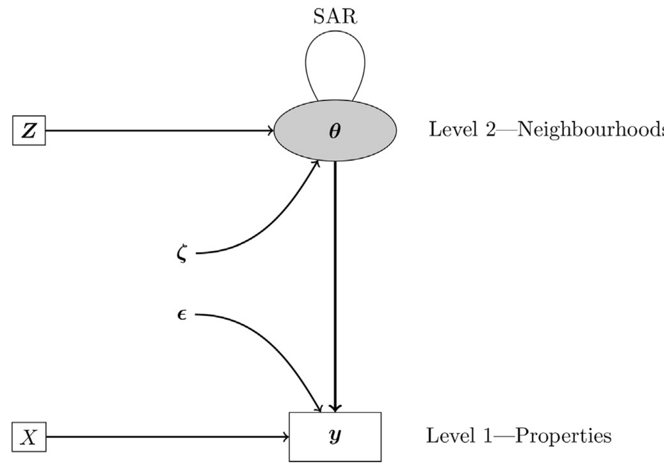

Fig. 1. The schematic diagram of a spatial multi-level model specified in Eqs. (2) and (3).

In the locally adaptive spatial multi-level model, Eq. 3 is re-formulated to,

$$\theta = \rho\widetilde{W}\theta + Z\boldsymbol{\gamma} + \zeta, \tag{4}$$

in which $\widetilde{W}$ is the final estimated W. Eqs. (2) and (4) complete the proposed locally adaptive spatial multi-level model. The spatial auto-regressive parameter ρ captures the strength of global spatial auto-regression in neighbourhood quality while each element of $\widetilde{W}$, $\widetilde{w}_{lk}$ specifically determines whether or not neighbourhoods (l, k) may be conditionally dependent. Following Lee and Mitchell (2013), when $\widetilde{w}_{lk} = 0$ and $w_{lk} = 1$,a boundary or step change between (l, k) is identified.

2.2. Estimation for the locally adaptive spatial multi-level model

An iterative algorithm is employed to implement the locally adaptive spatial multi-level model, an estimation strategy that has been used in the contexts of a single-level spatial statistics model (Lee & Mitchell, 2013) and a spatio-temporal statistics model (Lee & Mitchell, 2014). In this estimation strategy, model parameters are spilt into two sets: $\Theta = [\boldsymbol{\beta}, \gamma, \boldsymbol{\theta}, \rho, {\sigma_\epsilon}^2, {\sigma_\zeta}^2]$ and binary quantities in W. We note that changes to the elements of W only consider trimming a connection, changing its representation from 1 to 0 in a subsequent W. No additional links may be admitted, so W is the upper bound of connectivity in the problem. The algorithm iterates between updating Θ given W, $f(\Theta | W, y, X, Z)$, and updating W conditioning on Θ, $f(W | \Theta, y, X, Z)$, until a convergence criterion is met. The estimation of $f(\Theta | W, y, X, Z)$ is done by using a Bayesian MCMC approach, which will be detailed in the following section.

The update of W is via a deterministic approach, relying on the empirical posterior samples of $\boldsymbol{\theta}$. For geographically adjacent neighbourhoods (l, k), w_{lk} is set to 0 if the $1 - \alpha$% credible intervals of θ_l and θ_k are not overlapping, and is kept as 1 otherwise. For this case study, we use $\alpha = .05$. In practice, if two adjacent neighbourhoods (k, l) have interval-distinct estimates ($\boldsymbol{\theta}_k$, $\boldsymbol{\theta}_l$), then they have a significantly different estimated quality in spite of their geographic connection. It is useful to note that having disjoint interval estimates does not mean the difference between the point estimates is substantively meaningful; large samples will pick up nuanced differences between neighbourhoods because the credible intervals may be quite precise. Regardless, with $\widetilde{W}$, other meaningful difference thresholds can be further imposed to select boundaries that are both statistically and substantively significant, as demonstrated in (Dean et al., 2018). The overall algorithm is briefly summarised as below.

Step 1—Initialise model estimation. Estimating starting values of Θ by assuming neighbourhood quality $\boldsymbol{\theta}$ to be spatially independent, labelled as $f(\Theta^{(0)}| W,y,X,Z)$.

Step 2—Iterate the estimation of $f(\Theta| W,y,X,Z)$ and $f(W|\Theta,y,X,Z)$. (a) Prune W to estimate $W^{(t+1)}$ based on the posterior distributions of $\boldsymbol{\theta}^{(t)}$. To do this, set $w_{lk}{}^{(t+1)} = 0$ if the marginal 95% credible intervals of $\theta_l{}^{(t)}$ and $\theta_k{}^{(t)}$ are disjoint and $w_{lk} = 1$. Otherwise, retain w_{lk}. Note that this operation modifies W and not $W^{(t)}$, so $W^{(t+1)}$ may be more dense than $W^{(t)}$, but can never be denser than W. (b) Estimate $f(\Theta^{(t+1)}| W^{(t)},y,X,Z)$ based on the Bayesian spatial multi-level model specified in Eqs 2 and 3 using $W^{(t+1)}$, the updated spatial weights matrix estimate.

Step 3—Terminate the iteration process. This can occur when one of two termination criteria are reached. The first is when W stops changing, i.e. $W^{(t^*+1)} = W^{(t^*)}$. The second is when W cycles over k different states, such as $(W^{(t^*)}, W^{(t^*+1)}, \ldots, W^{(t^*+k-1)}, W^{(t^*)}, \ldots)$. Should this happen, the W in this cycle that yields the smallest Moran's I for $\hat{\theta}$ is chosen as the final spatial weights matrix, $\widetilde{W}$.

Step 4—Estimate the final spatial multi-level based on $\widetilde{W}$.

The convergence of W is guaranteed as the sampling space for W is finite despite of its large size of $2^{1'}W1/2$ (Lee & Mitchell, 2013). In addition, the first termination criterion is met in most cases.

2.3. Bayesian MCMC estimation of a spatial multi-level model

Before describing the estimation of the spatial multi-level model, we illustrate the interpretation of regression coefficients of predictors at the property and neighbourhood scales. The partial effect of an neighbourhood-scale predictor (e.g. z_k) on the latent neighbourhood quality $\boldsymbol{\theta}$ is expressed as,

$$\frac{\partial\theta}{\partial z_k} = (I_J - \rho\widetilde{W})^{-1}\gamma_k. \tag{5}$$

The effect z_k on $\boldsymbol{\theta}$ therefore can be interpreted in terms of direct, indirect and total impacts following (Elhorst, 2010; LeSage & Pace, 2009). The direct impact of z_k is calculated as the average of the diagonal elements of Eq. (5), i.e. $\gamma_k trace((I_J - \rho\widetilde{W})^{-1})$. The total impact of z_k is $\gamma_k/(1-\rho)$ while the indirect impact is the difference between total impact and direct impact. The neighbourhood quality impacts of a one unit change in z_k are then passed on to changes in property prices. At the property scale, the partial effect of a predictor (e.g. x_p) on property price is its coefficient β_p, holding other variables constant.

We now describe the Bayesian MCMC estimation of the spatial multi-level model, as specified in Eqs. (2) and (3). The joint distribution of model parameters Θ is proportional to the product of data likelihood $f(\boldsymbol{y}|.)$ and prior densities specified for them $p(.)$, as expressed below

$$f(\Theta \mid W,\boldsymbol{y},X,Z) \propto f(\boldsymbol{y}\mid\Theta)p(\theta\mid\rho,\sigma_\zeta^2,\boldsymbol{\gamma})p(\rho)p(\sigma_\zeta^2)p(\sigma_\epsilon^2)p(\boldsymbol{\beta})p(\boldsymbol{\gamma}). \tag{6}$$

The prior distributions for regression coefficients and variance parameters were assumed to be independent and specified following the conventions in the Bayesian spatial econometrics and multi-level modelling literature (e.g. Gelman et al., 2014; LeSage & Pace, 2009). More specifically, the property-level regression coefficients $\boldsymbol{\beta}$ follows a multivariate Normal distribution with mean M_0 and variance matrix T_0, $p(\boldsymbol{\beta}) \sim MVN(M_0,T_0)$, and for the neighbourhood-level regression coefficients γ, $p(\gamma) \sim MVN(M_1,T_1)$. We assign a uniform prior to ρ over $(-1,1)$, thus allowing for the possibility of a negative spatial autocorrelation. Inverse Gamma distributions (IG) are specified for the two variance parameters $\sigma_\epsilon{}^2$ and $\sigma_\epsilon{}^2$; $p(\sigma_\epsilon{}^2) \sim IG(a_0,b_0)$ and $p(\sigma_\zeta{}^2) \sim IG(a_1,b_1)$ with a and b being the shape and scale parameters, respectively.

The likelihood function of the model is expressed as,

$$f(\Theta) = (2\pi\sigma_\epsilon^2)^{-N/2}\exp\{-0.5\sigma_\epsilon^{-2}(\boldsymbol{y}-X\boldsymbol{\beta}-\Delta\theta)^T(\boldsymbol{y}-X\boldsymbol{\beta}-\Delta\theta)\}. \tag{7}$$

Based on Eq. (3) and using the Jacobian method (transforming the spatially dependent $\boldsymbol{\theta}$ to an independent vector, Anselin, 1988). The prior distribution $p(\boldsymbol{\theta}|\rho,\sigma_\zeta{}^2)$ is

$$p(\theta\mid\rho,\sigma_\zeta^2) = |A|(2\pi\sigma_\zeta^2)^{-J/2}\exp\{-0.5\sigma_\zeta^{-2}(A\theta-Z\gamma)^T(A\theta-Z\gamma)\}, \tag{8}$$

where $A = I_J - \rho W$ and $|A|$ is the absolute value of the determinant of A.

Combining Eq. (7) and prior distributions yields the conditional posterior distributions for model parameters. The conditional posterior distribution for the property-scale regression coefficients $\boldsymbol{\theta}$ is also a multivariate Normal distribution, $f(\boldsymbol{\beta}|.) \sim MVN(M_{\boldsymbol{\beta}},\Sigma_{\boldsymbol{\beta}})$ with

$$\Sigma_\beta = (X^TX/\sigma_\epsilon^2 + T_0^{-1})^{-1}; \quad M_\beta = \Sigma_\beta[X^T(\boldsymbol{y}-\Delta\theta) + T_0^{-1}M_0]. \tag{9}$$

The conditional posterior distribution for the latent neighbourhood quality $f(\boldsymbol{\theta}|.)$ is a multivariate Normal distribution, $MVN(M_{\boldsymbol{\theta}},\Sigma_{\boldsymbol{\theta}})$ with

$$\Sigma_\theta = (\Delta^T\Delta/\sigma_\epsilon^2 + A^TA/\sigma_\zeta^2)^{-1}; \quad M_\theta = \Sigma_\theta[\Delta^T(\boldsymbol{y}-X\boldsymbol{\beta}) + A^TZ\boldsymbol{\gamma}/\sigma_\zeta^2]. \tag{10}$$

With $\boldsymbol{\theta}$ having been sampled, it is treated as a dependent variable to draw the conditional posterior distribution of regression coefficients γ at the neighbourhood scale. As $f(\gamma|.) \propto p(\boldsymbol{\theta}|\rho,\sigma_\zeta{}^2,\gamma)p(\gamma)$, it is a multivariate Normal distribution, $MVN(M_\gamma,\Sigma_\gamma)$ with

$$\Sigma_\gamma = (Z^TZ/\sigma_\zeta^2 + T_1^{-1})^{-1}; \quad M_\gamma = \Sigma_\gamma[Z^TA\theta/\sigma_\zeta^2 + T_1^{-1}M_1]. \tag{11}$$

The posterior distributions for the two variance parameters are both Inverse Gamma: $f(\sigma_\epsilon{}^2|.) \sim IG(a_\epsilon,b_\epsilon)$ and $f(\sigma_\zeta{}^2|.) \sim IG(a_\zeta,b_\zeta)$, in which

$$a_\epsilon = N/2 + a_0; \quad b_\epsilon = b_0 + 0.5(\boldsymbol{y}-X\boldsymbol{\beta}-\Delta\theta)^T(\boldsymbol{y}-X\boldsymbol{\beta}-\Delta\theta) \tag{12}$$

$$a_\zeta = J/2 + a_1; \quad b_\zeta = b_1 + 0.5(A\theta-Z\boldsymbol{\gamma})^T(A\theta-Z\boldsymbol{\gamma}) \tag{13}$$

The conditional posterior distribution of the spatial autoregressive parameter ρ is expressed as,

$$\begin{aligned} f(\rho\mid.) &\propto p(\theta\mid\rho,\sigma_\zeta^2,\boldsymbol{\gamma})p(\rho) \\ &\propto |I_J-\rho W|\exp\{-0.5\sigma_\zeta^{-2}(A\theta-Z\boldsymbol{\gamma})^T(A\theta-Z\boldsymbol{\gamma})\} \end{aligned} \tag{14}$$

This is not a commonly-recognized probability density function, so a direct Gibbs sampler is not directly applicable (Gelman et al., 2014). Following prior studies on spatial multi-level model development (e.g. Dong & Harris, 2015), an inverse sampling approach is employed for the posterior inference on ρ. The approach starts by empirically evaluating the log-posterior density function of ρ, $\log f(\rho|.)$, based on the updated values of $(\boldsymbol{\theta}^{(r)},\boldsymbol{\beta}^{(r)},\gamma^{(r)},\sigma_\epsilon{}^{2(r)},\sigma_\zeta{}^{2(r)})$ in the rth MCMC iteration. Using this strategy, $\log f(\rho|.)$ is evaluated as,

$$\log f(\rho\mid.) = \log|I_J-\rho W| - (e_0^{(r)}-\rho e_d^{(r)})^T(e_0^{(r)}-\rho e_d^{(r)})/2\sigma_\zeta^{2(r)} + C \tag{15}$$

$$e_0^{(r)} = \theta^{(r)}(I_J - Z(Z^TZ)^{-1}Z); \; e_d^{(r)} = W\theta^{(r)}(I_J - Z(Z^TZ)^{-1}Z). \tag{16}$$

In the above formulas, C is a constant or an normalised density term. e_0 and e_d are two column vectors of residuals when regressing $\boldsymbol{\theta}^{(r)}$ and $W\boldsymbol{\theta}^{(r)}$ on neighbourhood-level independent variables Z. We then numerically integrate $\log f(\rho|.)$ over the feasible range of $\rho(-1,1)$, calculate the empirical cumulative distribution, and draw samples of $\rho^{(r)}$.[1]

[1] The evaluation of $\log f(\rho|.)$ takes the updated values of other model parameters as known inputs. Expanding the term $(A\boldsymbol{\theta} - Z\gamma)$ in Eq. (14), we get $(I_J - \rho W)\boldsymbol{\theta} - Z\gamma$. With known ρ, the term $(I_J - \rho W)$ transforms the spatially dependent neighbourhood quality $\boldsymbol{\theta}$ to an independent variable, as indicated by Eq. (3). As such, $(A\boldsymbol{\theta} - Z\gamma)$ is simply the model residual term when regressing the transformed $\boldsymbol{\theta}$ on Z, with γ being the ordinary least squares

The above MCMC samplers and the overall iterative estimation algorithm for the locally adaptive spatial multi-level model are coded by using the R language (R Core Team, 2017) and available in the Supplementary Online Materials associated with the paper. In the following analyses of property prices, statistical inferences on model parameters are based on two MCMC chains, each consisting of 10,000 iterations with a burn-in period of 5000. Convergence of samplers is checked by both visual inspection of trace plots of parameters and the Brooks-Gelman-Rubin scale reduction statistics (Brooks & Gelman, 1998; Gelman et al., 2014).

3. Data and variables

The study primarily draws upon individual property transaction records, made publicly available in the UK through the HM Land Registry. The Land Registry gathers information about property transactions in England and Wales on a monthly basis since 1995 and compiles a database called Price Paid Data (PPD).[2] The PPD contains a few property-level characteristics, including property address, transaction price, type and dates. In this study, we extracted all property transactions from 2010 to 2015 for the Liverpool Local Authority District (LAD). The dataset was subsequently cleaned to include only properties sold for full market values, since property transfers through repossessions or buy-to-lets seldom reflect real property market values. This leads to a final sample of 26,468 property transaction records used in the following analyses. The sample mean property price is £132,900 with a standard deviation of £ 90,460. To reduce potential heteroskedasticity impacts on model estimates, property prices were log-transformed in all models.

A fine-resolution census geography, Lower Layer Super Output Area (LSOA), was used as neighbourhood units in the study. LSOA serves as the main geography through which the Office for National Statistics and other government departments in the UK release small area statistics. Based on the 2011 census data, LSOAs in Liverpool LAD have an average population of about 1565 with a standard deviation of about 296. The LSOA (neighbourhood) boundaries in Liverpool was shown in Fig. 2, with colour shaded based on the quintiles of LSOA-scale average property prices. A clear spatial clustering pattern of the average property prices appears. This produces a Moran's I statistic of 0.699 (p value < 0.001), indicating a significant positive spatial auto-correlation in neighbourhood-scale average prices. Equally clear in Fig. 2 are spatial discontinuities or step changes in the distribution of property prices: pairs of neighbourhoods sharing geographical borders are however associated with contrasting property prices. These features revealed by the simple choropleth map implies the necessity of capturing both global spatial auto-regression and local step changes when modelling geographical variables.

The independent variables were extracted at the property and neighbourhood scales. Property characteristics included in the model are property types (detached, semi-detached, terrace or flats) and tenure status (leasehold versus freehold). To control for temporal fluctuations in the overall housing markets of Liverpool during the study period, both year and month dummy variables are included in our model. At the neighbourhood scale, a set of social, ethnic, built environment, and locational variables were linked to neighbourhood quality, following prior hedonic price literature (e.g. Anselin & Le Gallo, 2006; Dubin, 1992; Lazrak et al., 2014). They include proportions of non-white British population, unemployment rates, and

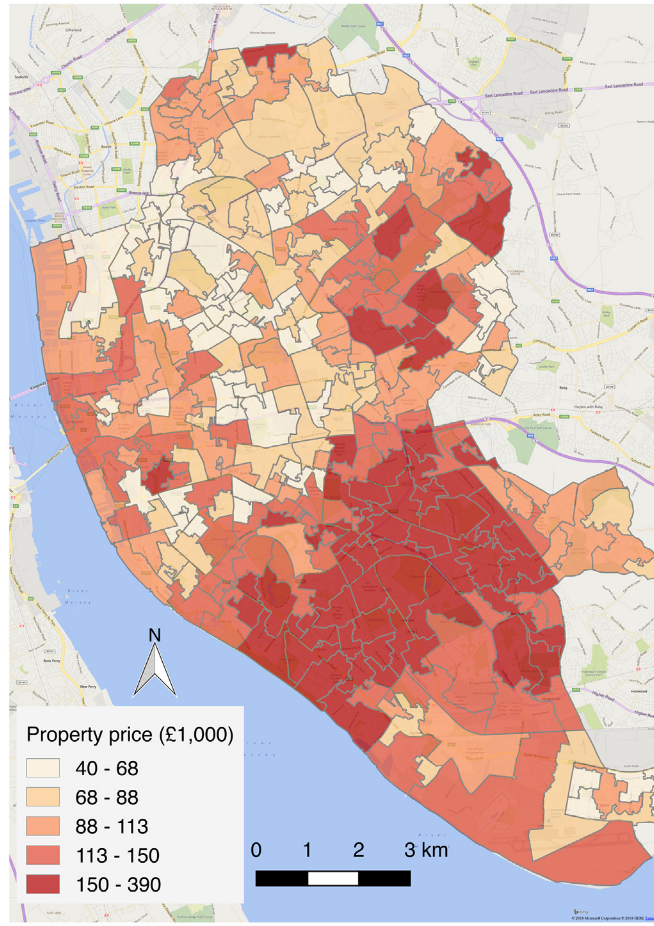

Fig. 2. The spatial distribution of neighbourhood (LSOA) scale property prices.

population density of each LSOA extracted from the 2011 census data; crime counts in each LSOA during the study period calculated by using the open UK policing data portal (https://data.police.uk/); green space areas provided by the Ordnance Survey-Greenspace product; the number of bus stops in each LSOA calculated by using the National Public Transport Access Nodes (NaPTAN) database; geographical proximity to the nearest retail centre publicly available from the Consumer Research Data Centre (CDRC); and geographical proximity to the nearest primary schools and General Practice (GP) extracted from the English index of multiple deprivation in 2015. All these data are publicly available and also provided in the Supplementary Online Materials associated with the paper. Summary statistics of variables included in the study are displayed in Table 1.

4. Results and discussions

Table 2 displays estimation results for both the locally adaptive and global spatial multi-level models. Before interpreting the estimated geography of neighbourhood quality and covariate effects, we first discuss the comparison of the two model specifications and its implications. Deviance Information Criterion (DIC, Spiegelhalter, Best, Carlin, & Van Der Linde, 2002), a commonly used model fit index in Bayesian inference, has been calculated for each model. A better model specification is indicated by a smaller DIC value. A decrease of 6 in DIC values from the non-adaptive to the locally adaptive spatial multi-level model provides evidence on the improved model fit.

The estimated global spatial autoregressive parameter ρ is slightly larger in the non-adaptive spatial multi-level model than in the adaptive model. This is driven by the step changes identified in the neighbourhood quality surface, since quite a few neighbourhoods are

(*footnote continued*)
estimator—$\gamma = (Z^T Z)^{-1} Z(\boldsymbol{\theta} - \rho W \boldsymbol{\theta})$. Substituting γ with its estimator in $(A\boldsymbol{\theta} - Z\gamma)$ gives Eq. (16) in the main text.

[2] Contains HM Land Registry data ©Crown copyright and database right 2017. This data is licensed under the Open Government License v3.0. All the data used in the study is under the same license.

Table 1
Statistical summaries on data and variables.

Variable names	Description	Mean/ proportions
Log Price	Log of property transaction prices	11.6 (0.6)
Property-level independent variables		
Property type	Detached	8.6%
	Semi-detached	26.1%
	Flat	20.6%
	Terrace	44.6%
Tenure	Freehold	63.5%
	Leasehold	36.5%
Time	The month and year of a property transaction	
Neighbourhood-level independent variables		
Non-white	Proportion of non-white British population	0.10 (0.11)
Unemployment	Unemployment rate	0.11 (0.03)
Population density	Log of population density (persons per square kilometer)	4.28 (0.46)
Crime	Log of crime counts	5.12 (0.72)
Green space	Log of green areas in each LSOA	7.30 (4.71)
Primary school	Log of distance to the nearest primary school	6.49 (0.43)
GP	Log of distance to the nearest General Practice	6.60 (0.43)
Retail centre	Log of distance to the nearest retail centre	6.74 (0.76)
Bus	Log of the number of bus stops per 1000 persons	1.51 (0.63)
N	Number of property transactions	26,468
J	Number of neighbourhoods (LSOA)	298

Table 2
Estimation results from spatial multi-level models.

	Locally adaptive model			Non-adaptive model		
Variables	Median	2.5%	97.5%	Median	2.5%	97.5%
Property-level independent variables *X*						
Intercept	12.17[a]	12.08	12.23	12.39[a]	12.29	12.47
Flat	−0.965[a]	−0.988	−0.943	−0.968[a]	−0.991	−0.944
Semi-detached	−0.380[a]	−0.397	−0.363	−0.382[a]	−0.399	−0.365
Terrace	−0.643[a]	−0.660	−0.625	−0.645[a]	−0.663	−0.627
Leasehold	0.001	−0.012	0.013	0.001	−0.014	0.012
Year dummy variables	YES	YES				
Month dummy variables	YES	YES				
Neighbourhood-level independent variables *Z*						
Intercept	0.790[a]	0.224	1.385	0.681[a]	0.069	1.271
Non-white	0.70[a]	0.348	1.058	0.803[a]	0.439	1.172
Non-white squared	−1.287[a]	−2.103	−0.440	−1.742[a]	−2.622	−0.834
Unemployment	−4.007[a]	−4.894	−3.189	−3.273[a]	−4.220	−2.369
Population density	−0.060[a]	−0.119	−0.003	−0.094[a]	−0.154	−0.031
Crime	−0.040[a]	−0.078	−0.002	−0.038	−0.080	0.004
Green space	0.001	−0.003	0.006	0.001	−0.004	0.006
Primary school	0.012	−0.037	0.059	0.021	−0.030	0.070
GP	−0.057[a]	−0.104	−0.009	−0.022	−0.071	0.026
Retail centre	0.019	−0.010	0.047	0.001	−0.029	0.031
Bus	0.056[a]	0.016	0.097	0.046[a]	0.002	0.088
σ_ϵ^2	0.107	0.105	0.108	0.107	0.105	0.108
σ_ζ^2	0.025	0.020	0.030	0.028	0.023	0.033
ρ	0.635	0.572	0.693	0.712	0.638	0.782
DIC	16,152			16,158		

[a] Represent statistical significance at the 95% credible interval.

disconnected from nearby neighbourhoods over the course of iterations. The statistical significance of ρ in both models demonstrates the necessity of taking the spatial autoregressive effect into account when using multi-level models of explicitly geographical processes.

The geography of estimated neighbourhood quality ($\boldsymbol{\theta}$) is illustrated in Fig. 3, divided into six categories using the natural break scheme. There are 1642 pairs of geographically continuous neighbourhoods (LSOAs sharing common borders), of which 1006 pairs are associated with statistically significant differences in the estimated neighbourhood quality. A further difference threshold (also discussed in Dean et al., 2018), the mean of the distribution of border-paired absolute differences of neighbourhood quality plus one standard deviation, is enforced for the identification of step changes. The left panel of Fig. 3 displays these step changes. The global clustering pattern of neighbourhood quality as measured by average house sale prices controlling for property-level characteristics is clear: the middle south of Liverpool and the waterfront area are places with good neighbourhood quality while North and South Liverpool are with poor neighbourhood quality.

An important feature of these step changes or boundaries is that many of them are not enclosed – a LSOA could be substantially different from some of its geographic neighbours in certain directions but blend into others. Step changes can be also classified by magnitude. For instance, if the difference on the opposite sides of a boundary was larger than the mean of all boundary-pair differences, it could be referred to as a hard boundary, and a moderate boundary otherwise, as depicted in the right panel of Fig. 3. Many of these hard boundaries are concentrated along the waterside of Liverpool to the river Mersey. Liverpool's waterfront area has been under a regeneration scheme involving replacement of the old dock areas with residential buildings and cultural and recreational amenities. The area exhibiting high property values extending from the waterfront inwards represents the Liverpool Georgian Quarter, an area historically occupied by affluent merchants that has been revitalised recently. The model clearly identifies the division between these two sections of the city turned around by recent investment and regeneration, and areas where funding has been much scarcer, such as the neighbourhood of Toxteth, immediately located on the south of the Georgian Quarter (Sykes, Brown, Cocks, Shaw, & Couch, 2013). Toxteth's southern border is also picked up by the model to differentiate it from the more affluent suburbs starting directly to its south. Equally evident are abrupt changes scattering across the study area and within the areas with good neighbourhood quality. In general, areas where the local mix of land uses diverges from the surrounding areas (e.g. as a result of public policy, infrastructure or new development) tends to produce boundaries. The locality where step changes take place might be due to multiple reasons including large spatial gradients in neighbourhood characteristics, physical environment, urban infrastructure and so on. It is possible that mechanisms of boundary formation might vary across space, requiring in-depth field work to be conducted to understand the nuanced geography of neighbourhood quality in Liverpool, which is beyond the scope of the current research.

With respect to property-level characteristics, estimates on their coefficients in the two models are much similar. Prices of detached properties are on average 38%, 64.3% and 96.5% higher than that of semi-detached properties, terrace properties and flats, respectively. Property tenure is not found to be significantly related to prices, after controlling for property types and time (both year and month) fixed effects.

Next, we turn to estimates on regression coefficients of neighbourhood characteristics. In terms of population ethnic composition, a non-linear association between proportions of non-white British population and neighbourhood quality is found, ceteris paribus. Neighbourhood quality tends to first increase with increasing non-white British population shares until a point where the proportion reaches about 37.6% ($0.5 \times 0.7/1.287 + 0.104$), and then declines with further increases of non-white population. This raises a question in relation to how property market and people's residence choices response to the concentration of ethnic minorities or population ethnic integration at a fine spatial scale. Similar non-linear effects of ethnic minority concentration on neighbourhood population dynamics have been found in the US context (e.g.

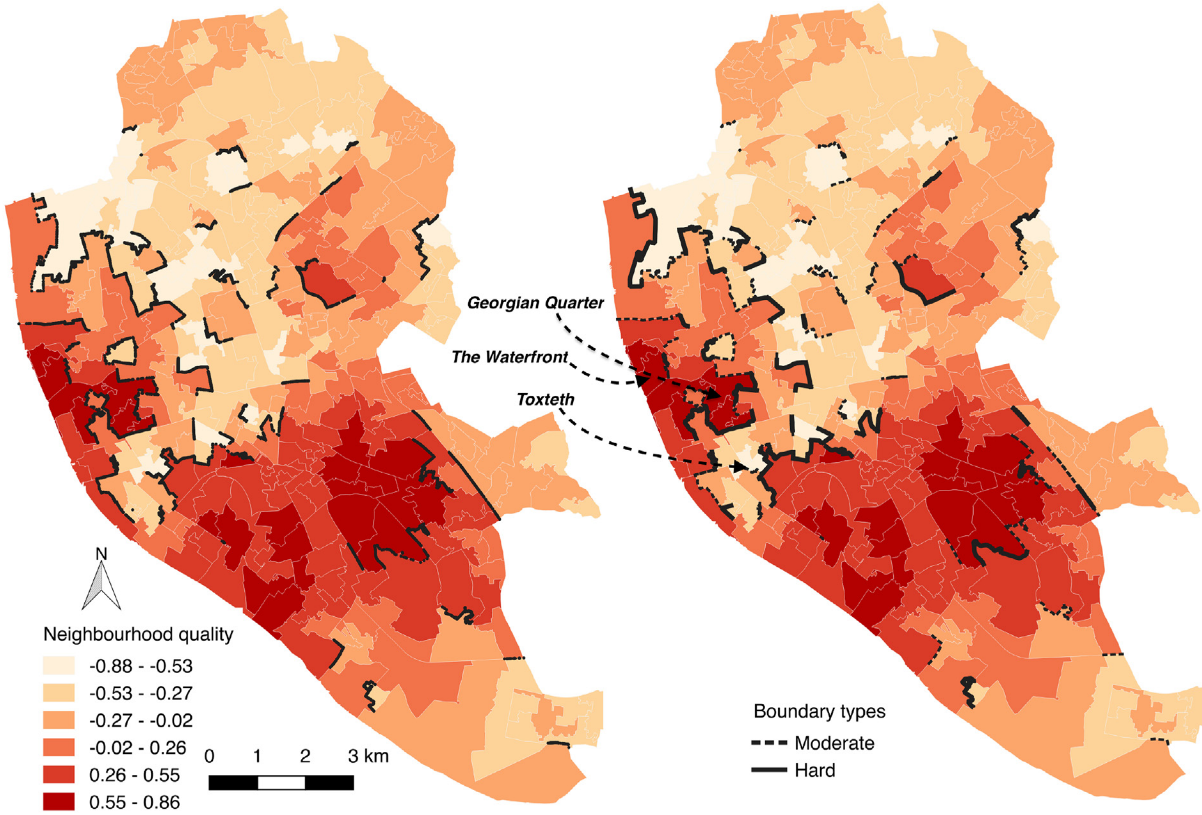

Fig. 3. The geography of neighbourhood quality in Liverpool, superimposed by estimated step changes or boundaries.

Card, Mas, & Rothstein, 2008). As expected, larger unemployment rate and crime prevalence are associated with lower neighbourhood quality, everything else equal. Population density is negatively related to neighbourhood quality due to possible congestion effects or high competition for resources at a local scale. At a more aggregated spatial scale such as cities, higher population density tends to be more linked to higher labour productivity and various agglomeration economies than negative externalities, thus positively linked to average property prices (e.g. Glaeser, Gyourko, & Saks, 2005).

Further, geographical accessibility to both public transport and health care facilities (e.g. GP) are associated with elevated neighbourhood quality. However, the association between primary school proximity and neighbourhood quality is not significant in this study. One reason might be the partial role of geographical proximity in the complex delineation of school catchment areas in the English school admission system (Harris, Johnston, & Burgess, 2016; Singleton, Longley, Allen, & OBrien, 2011). Proximity to local retail centres and the proportion of green areas of each LSOA are also not significantly related to neighbourhood quality, after controlling for other neighbourhood characteristics.

Comparing estimates on neighbourhood-level independent variables from the two models, some interesting differences with respect to inference on statistical significance were spotted. For instance, crime prevalence and proximity to GP are not statistically significantly associated with neighbourhood quality in a non-adaptive spatial multi-level model. They become significant after capturing the local steps changes or discontinuities of neighbourhood quality in the preferred locally adaptive spatial multi-level model. Although our primary interest is not on exploring the true effects of crime and proximity to health care on neighbourhood quality, differences in the statistical significance of effects under changes in spatial structure highlight the importance of a proper treatment of spatial dependence or group dependence, as is the case elsewhere in the literature (Hodges & Reich, 2010).

A range of neighbourhood socio-demographic, economic and geographical accessibility characteristics have been included in our model to explain neighbourhood quality variations. This is, by no means, an exhaustive list. Nor is our purpose to enumerate all relevant factors contributing to neighbourhood quality. The estimates on coefficients of neighbourhood-scale variables in the developed methodology take into account the discontinuity in neighbourhood quality ($\boldsymbol{\theta}$), thus being more reliable than that from a global spatial multi-level model that only captures large-scale smoothness in $\boldsymbol{\theta}$.

5. Conclusion

This study developed a locally adaptive spatial multi-level model, drawing upon a recently proposed spatial statistics methodology that estimates the correlation structure among spatial units and regression coefficients at the same time. The methodology provides a unified framework for simultaneously modelling global or overall spatial dependence, local discontinuities or step changes, and group dependency effects such that multiple-scale spatial data could be better investigated. Further, the general idea of iteratively updating spatial correlation structures and estimating model parameters in a locally adaptive spatial multi-level model can be generalised to many more standard spatial econometrics models familiar to spatial analysts. The methodology also produces intuitive interpretation on regression coefficients for independent variables at different scales or levels. In short, higher-level covariates affect the outcome variables measured for lower-level units indirectly through a latent higher-level outcome variable whilst lower-level covariates are linked to the outcome variable directly. This avoids the conflation of effects on the outcome variables from covariates at different scales.

Based on individual property transactions provided in the Land Registry data, we applied the developed methodology to infer neighbourhood quality and explore its potential correlates. The derived geography of neighbourhood quality in Liverpool shows interesting features. It depicts a clear spatial clustering pattern of neighbourhood quality. Meanwhile, local step changes are observed across Liverpool, especially within the cluster of neighbourhoods with good quality surrounding the city centre. The co-existence of a global smoothness trend and significant local departures demonstrates a complex and fragmented geography of neighbourhood quality, which makes a simplified abstraction of the distribution of neighbourhood quality as a global autoregressive process inappropriate. Estimates on the geography of neighbourhood quality in Liverpool can be disseminated as useful urban indicators at a fine-resolution spatial scale, which would be of great potential to inform residents' housing choices and local government's urban (re)generation and development policies.

Neighbourhood socio-demographic, economic and built environment characteristics are statistically significant determinants of neighbourhood quality. Suitable levels of ethnicity integration, low unemployment rate and a good social security promotes good neighbourhood quality. In addition, low population density and high geographical accessibility to public transport and health care are positively related to neighbourhood quality. The differences in estimates on coefficients of neighbourhood characteristics variables and, more importantly, their statistical inferences between the adaptive and non-adaptive models highlight the importance of taking into account local step changes when modelling spatial dependence.

Some limitations remain in the study. The first is in relation to the property transaction data from Land Registry. Property size information is not available in the data, which might have an impact on the estimated neighbourhood quality. A promising way to address this limitation is to link property transaction records from Land Registry with property energy performance data recently made available to researchers. Our future work on neighbourhood quality estimation would be based on the linked property transaction data. The second limitation is related to our methodological development. Potential spatial dependence among properties is not explicitly modelled in the study. The key reason is computational – further incorporation of spatial dependence at the property level is likely to make the estimation of a locally adaptive spatial multi-level model impractical because of the large sample size. However, both group dependence between properties located in the same neighbourhood and spatial dependence among neighbourhoods are captured in our model. Since neighbourhoods are approximated by spatial units with fine granularity, we would expect the influence on our model estimation results of property-level spatial dependence to be insignificant. Lastly, potential temporal dependence effect at the neighbourhood-scale model is not captured in the developed methodology. An important avenue for future work is to incorporate temporal auto-correlation into the current model so that temporal dynamics in neighbourhood quality could be investigated.

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